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ON THE ENERGETIC EFFICIENCY OF SOME MECHANICAL TRANSMISSIONS

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Abstract: In the paper, it is determined the energetic efficiency for three types of industrial equipments which, in addition to the base component elements – the driving motor and the working machine - contain flywheels and complex transmissions, composed by hydraulic and electromagnetic couplings, gear boxes and speed variators. These kinds of complex transmissions are used, especially, in the metallurgical industry.

1. FIRST INDUSTRIAL EQUIPMENT

It is considered the industrial equipment, whose dynamic model has the structure in figure 1, [1], where:

- I – driving motor;
- II – gear box;
- III – speed variator;
- IV – flywheel;
- V – working machine.

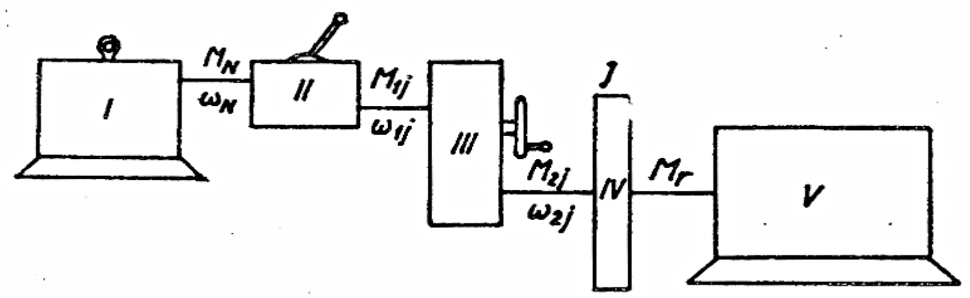

Fig.* 1. *Dynamic model of the first industrial equipment

The electric motor, of asynchronous type, works in nominal regime, [2], i. e. it turns with the nominal angular velocity ω_N and generates the nominal torque of moment M_N. The gear box modifies in steps, the angular velocity of motor. When in the gear box the *j* speed is coupled, the torque and angular velocity at the driven shaft are M_{1j}, respectively ω_{1j}, and

$$M_{1j}\omega_{1j} = \eta_{cvj} M_N \omega_N , \tag{1}$$

where η_{cvj} is the energetic efficiency of the gear box, in the *j* speed.

Between two gear speeds, the angular velocity is continuously adjusted with the help of the speed variator. The energetic efficiency of the speed variator depends on the input angular velocity ω_{1j} and ratio of transmission *i*:

$$\eta_{vj} = \eta_{vj}(\omega_{1j}, i) . \tag{2}$$

The torque and angular velocity at the driven shaft of the speed variator are

$$M_{2j}\omega_{2j} = \eta_{vj}(\omega_{1j}, i) M_{1j}\omega_{1j} , \tag{3}$$

where:

$$i = \frac{\omega_{1j}}{\omega_{2j}} . \tag{4}$$

On the driven shaft of the speed variator, having an angular velocity, adjustable in steps, and continuously adjusted between the speed steps, there are mounted the working machine which creates the resistant moment M_r and the flywheel whose presence limits the degree of non uniformity of motion.

The energetic efficiency of transmission of the industrial equipment is

$$\eta = \frac{E_c}{L} , \tag{5}$$

where E_c is the kinetic energy, transferred to the working machine and flywheel, during the period of acceleration until the angular velocity of regime, and *L* is the work, effected by the motor, with this aim.

The kinetic energy has the expression

$$E_c = \sum_{j=1}^{n} M_{2j}\theta_{2j} - M_r \sum_{j=1}^{n} \theta_{2j} , \tag{6}$$

where θ_{2j} is the angle, covered by the working machine and flywheel in the *j* speed, and *n* is the number of speed steps in the gear box.

From (3) it results

$$M_{2j} = \eta_{vj}(\omega_{1j}, i) \frac{M_{1j}\omega_{1j}}{\omega_{2j}} . \tag{7}$$

By applying the theorem of kinetic energy for the motion of the working machine and flywheel, between the speed steps $j-1$ and *j*, it is obtained

$$J \frac{\omega_{2j}^2}{2} - E_{c_{j-1}} = (M_{2j} - M_r)\theta_{2j} , \tag{8}$$

were J is the reduced moment of inertia of flywheel, the reduced moment of inertia of the working machine being, in general, negligible, in relation to the one of the flywheel.

From (8) it results

$$\omega_{2j} = \sqrt{\frac{2(M_{2j} - M_r)}{J}\theta_{2j} + \frac{2E_{c_{j-1}}}{J}} . \tag{9}$$

Taking into account the relations (4), (7) and (9), the kinetic energy (6) becomes

$$E_c = \sum_{j=1}^{n} \eta_{vj}\left(\omega_{1j}, \frac{\omega_{1j}}{\sqrt{\frac{2(M_{2j} - M_r)}{J}\theta_{2j} + \frac{2E_{c_{j-1}}}{J}}}\right) \frac{M_{1j}\omega_{1j}}{\sqrt{\frac{2(M_{2j} - M_r)}{J}\theta_{2j} + \frac{2E_{c_{j-1}}}{J}}}\theta_{2j} - M_r \sum_{j=1}^{n}\theta_{2j} . \tag{10}$$

The work effected by the motor is

$$L = M_N\theta_1 = M_N\omega_N t^* , \tag{11}$$

where θ_1 is the angle of rotation of motor, during the period of acceleration, and t^* is the duration of acceleration.

Taking into account the relations (1), (10) and (11), the energetic efficiency of the industrial equipment, (5), gets the expression

$$\eta = \frac{\sum_{j=1}^{n} \eta_{cvj}\eta_{vj}\left(\omega_{1j}, \frac{\omega_{1j}}{\sqrt{\frac{2(M_{2j} - M_r)}{J}\theta_{2j} + \frac{2E_{c_{j-1}}}{J}}}\right) \frac{1}{\sqrt{\frac{2(M_{2j} - M_r)}{J}\theta_{2j} + \frac{2E_{c_{j-1}}}{J}}}\theta_{2j}}{t^*} - \tag{12}$$

$$- \frac{M_r}{M_N\omega_N t^*}\sum_{j=1}^{n}\theta_{2j} .$$

2. SECOND INDUSTRIAL EQUIPMENT

To continue, it is considered the industrial equipment, having the dynamic model in figure 2, [1].

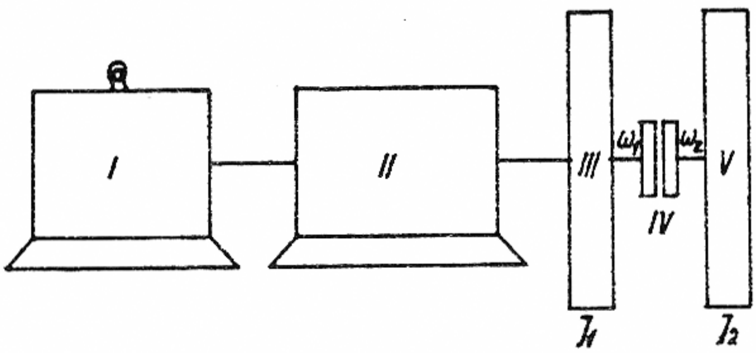

Fig. 2. Dynamic model of the second industrial equipment

The electric motor *I* directly drives the working machine *II* and the flywheel *III*, having the moment of inertia J_1; the reduced moments of inertia of the working machine and rotor of motor are negligible, in relation to the moment of inertia of the flywheel. The flywheel *III*, through a sliding coupling (hydraulic or electromagnetic) *IV*, puts in rotation motion a second flywheel *V*, with the moment of inertia J_2. When the sliding coupling is coupled, the second flywheel is accelerated until its angular velocity becomes equal to the one of the first flywheel. In the moment when the two angular velocities become equal, the coupling is decoupled. When the angular velocity of the flywheel *III* decreases under the one of the flywheel *V*, the coupling is automatically coupled and the flywheel *V* returns the kinetic energy to the main system, until the moment when the two angular velocities become equal again, and the coupling is decoupled again, too. Thus, the flywheel *V* works as a recuperator of kinetic energy. In this industrial equipment, in addition to the shafts which are rigid, there is a special element of transmission, the sliding coupling, placed between the two flywheels.

The energetic efficiency of the transmission in this case is

$$\eta = \frac{\Delta E_{c_2}}{\Delta E_{c_1}}, \tag{13}$$

where ΔE_{c_2} is the increasing of kinetic energy of the second flywheel, and ΔE_{c1} is the kinetic energy, consumed by the first flywheel, to accelerate the second one.

The increasing of kinetic energy of the second flywheel is

$$\Delta E_{c_2} = J_2 \frac{\omega_3^2 - \omega_2^2}{2}, \tag{14}$$

where ω_2 is the angular velocity of this flywheel in the moment when the sliding coupling is coupled, and ω_3 is the common angular velocity of both flywheels in the moment when their angular velocities become equal.

The kinetic energy, consumed by the first flywheel is

$$\Delta E_{c_1} = J_1 \frac{\omega_1^2 - \omega_3^2}{2}, \tag{15}$$

where ω_1 is its angular velocity in the moment when the sliding coupling is coupled.

By applying the theorem of conservation of angular momentum, from the moment when the coupling is coupled, until the moment when it is decoupled, it is obtained

$$J_1\omega_1 + J_2\omega_2 = (J_1 + J_2)\omega_3, \tag{16}$$

from which it results:

$$\omega_3 = \frac{J_1\omega_1 + J_2\omega_2}{J_1 + J_2}. \tag{17}$$

Taking into account the relations (14), (15) and (17), the energetic efficiency (13) gets the expression:

$$\eta = \frac{J_2(J_1\omega_1 + J_2\omega_2)^2 - J_2\omega_2^2(J_1 + J_2)^2}{J_1\omega_1^2(J_1 + J_2)^2 - J_1(J_1\omega_1 + J_2\omega_2)^2}. \tag{18}$$

In the particular case when the two flywheels have the same moment of inertia $(J_1 = J_2)$, and the second flywheel is in rest in the moment when the coupling is coupled $(\omega_2 = 0)$, from the formula (18) it results $\eta = 0.33$.

3. THIRD INDUSTRIAL EQUIPMENT

Finally, it is considered the industrial equipment, with the dynamic model in figure 3, [1], where *I* is the driving electric motor, working in nominal regime; *II* is the gear box; *III* is a sliding coupling, hydraulic or electromagnetic; *IV* is the flywheel, having a very big moment of inertia *J*; *V* is the working machine, whose moment of inertia is negligible in relation to the moment of inertia of the flywheel.

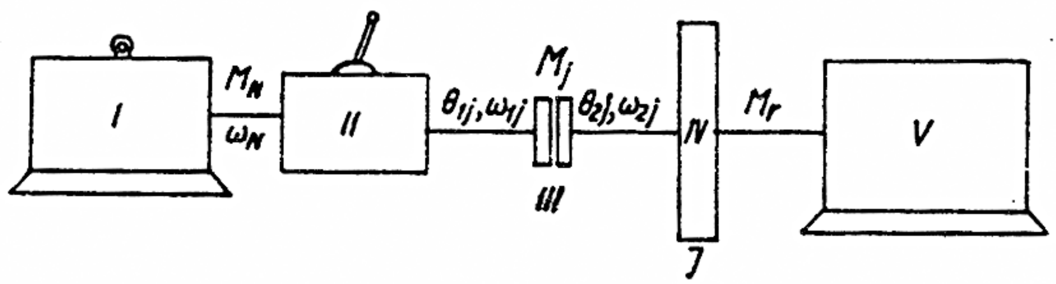

Fig. 3. Dynamic model of the third industrial equipment

The energetic efficiency of the sliding coupling, when in the gear box it is coupled the *j* speed step, and the coupling works with sliding, until the angular velocities ω_{1j} and ω_{2j} of half-couplings become equal, is

$$\eta_{ca_j} = \frac{E_{c_{2j}}}{L_{1_j}}, \tag{19}$$

where $E_{c_{2j}}$ is the kinetic energy, delivered to the flywheel an working machine in the *j* speed step, from the moment when the coupling is coupled, until the moment when the angular velocities of half-couplings become equal, and L_{1_j} is the work, effected by the driving half-coupling, in the same time interval.

If M_j is the moment of the torque, transmitted by the coupling in the *j* speed step, and θ_{1j} and θ_{2j} are the angles of rotation of the half-couplings in the *j* speed step, until the angular velocities of half-couplings become equal, the energetic efficiency (19) gets the expression:

$$\eta_{ca_j} = \frac{M_j\theta_{2j}}{M_j\theta_{1j}}. \tag{20}$$

In each j speed step, the driving half-coupling turns with the constant angular velocity ω_{1j}, so that the angle θ_{1j} is

$$\theta_{1j} = \omega_{1j} t_j , \tag{21}$$

where t_j is the time interval between the moment of coupling of the *j* speed step and the moment when the angular velocities of half-couplings become equal, i. e. the time interval when the sliding exists in the coupling. To determine t_j, it is applied the theorem of angular momentum for the motion of flywheel and working machine, during sliding in the coupling,

$$J\frac{d\omega_{2j}}{dt} = M_j - M_r , \tag{22}$$

where $\omega_{2j} = \omega_{2j}(t)$ is the angular velocity of the driven coupling, and $M_r = M_r(\omega_{2j})$ is the resistant torque of the working machine.

The torque M_j, transmitted by the coupling is a function of the sliding $\Delta\omega_j = \omega_{1j} - \omega_{2j}$ of coupling. From the relation (22), it results:

$$t_j = J\int_{\omega_{1,j-1}}^{\omega_{1j}} \frac{d\omega_{2j}}{M_j(\Delta\omega_j) - M_r(\omega_{2j})}. \tag{23}$$

Taking into account the relations (21) and (23), the energetic efficiency (20) becomes:

$$\eta_{ca_j} = \frac{\displaystyle\int_{\omega_{1,j-1}}^{\omega_{1j}} \frac{\omega_{2j} d\omega_{2j}}{M_j(\Delta\omega_j) - M_r(\omega_{2j})}}{\displaystyle\omega_{1j}\int_{\omega_{1,j-1}}^{\omega_{1j}} \frac{d\omega_{2j}}{M_j(\Delta\omega_j) - M_r(\omega_{2j})}}. \tag{24}$$

The total kinetic energy, spend to accelerate the flywheel and working machine, until the regime angular velocity is

$$E_{c_1} = \frac{J}{2}\sum_{j=1}^{n} \frac{\omega_{1j}^2 - \omega_{2,j-1}^2}{\eta_{ca_j}\eta_{cv_j}}, \tag{25}$$

where *n* is the number of speed steps in the gear box, and η_{cv_j} is the box energetic efficiency in the *j* speed step.

The kinetic energy of the flywheel and working machine when the regime angular velocity is reached is

$$E_{c2} = \frac{J\omega_{1n}^2}{2}. \tag{26}$$

The global energetic efficiency of energy transfer, from the motor to the working machine has the expression:

$$\eta = \frac{E_{c_2}}{E_{c_1}} = \omega_{1n}^2 \sum_{j=1}^{n} \frac{\eta_{ca_j}\eta_{cv_j}}{\omega_{1j}^2 - \omega_{2,j-1}^2}. \tag{27}$$

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